

Seabed Variability and Its Influence on Acoustic Prediction Uncertainty

Model and Data Variance and Resolution:

How Do We Quantify Uncertainty?

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LONG-TERM GOALS

A basic tenet of the Office of Naval Research's Uncertainty DRI is that, in any strategic situation, environmental parameters will never be known in complete enough detail to enable a perfectly accurate acoustic detection. In order to address the problem of unknown uncertainty this research is focused on two goals: 1. Assess and characterize seafloor variability in shelf areas. 2. Determine the impact of the seafloor variability on acoustic prediction uncertainty.

OBJECTIVES

The primary focus of this project will be to compute model and data sensitivities, investigate model parameter correlations, and optimal model parameterizations. The objectives can be stated as the answers to the following questions: Given the data we have available or can reasonably expect to be able to collect, which model parameters are most important? Can we resolve them? What will be the model variance? What additional data would be useful, if it were available? Which model parameters are essentially unresolvable (unconstrained)?

APPROACH

All our representations of the ocean/seafloor environment are, whether they are entries in a database or parameters supplied to a synthetic model, are under-parameterized versions of the true environment. Inverse problems that solve for environmental parameters are generally severely under-determined. This is because we attempt to represent what amount to continuous functions, e.g. water sound speed and bottom structure, with a finite number of discrete parameters. We hope that the parameters we choose capture most of the important features of the environment, but we are limited in the data we can collect by both practical considerations and physical constraints. Practically we can only collect a limited amount of data because of cost. Physical constraints make it impossible to obtain the kind of coverage available in a medical tomographic scan, for example. We simply do not have 360° coverage of our medium. Because of our limited data collecting capabilities and our finite model parameterization, we are forced to make trade-offs between variance reduction in our parameterized model and the resolution of our model (Menke, 1989).

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The issue is that our model has some variance, this is the model uncertainty, which we wish to reduce. We collect acoustic or other data, which has its own variances, and combine this data in an inverse problem to reduce the variance in whatever starting model we have chosen to represent the environment. A common approach is to search the model space for a set of parameters, which produce a model that fits the data according to some criterion like χ^2 statistic. Obtaining a model that fits the data is only half the problem. We still need to understand the variance and also the resolution of the model we have found. In fact the commonly used χ^2 statistic tells us nothing about the model variance, because it does not take the data variances into account. A low value of χ^2 tells us we have a model that fits the data, but it does not tell us how much of what we fit might be noise.

In addition to the model and data variance, we also need to know the resolution of both our data and our model. The variance and resolution are closely connected and we ultimately have to live with some compromise between the variance reduction in our model parameters and their resolution. We have a finite amount of data, some of which may be redundant. We can parameterize our environmental very finely, in which case we must use our limited data set to determine the values of many parameters. The result will be a model with many parameters, but with relatively large variances. On the other hand we could decide to parameterize our model very coarsely, and determine only a few parameters, with a consequent loss of resolution. However, because we have expended our data estimating only a few parameters, the variances of those parameters will be relatively lower. The choice of how to invest our data, whether to reduce the variance of a few parameters or to have a more finely parameterized model with larger variances, is ours to make.

It is possible to pre-compute measures of the model and data variance and resolutions. This allows us to determine the trade-offs available between the variance and resolution. The model and data variance and resolution matrices depend on partial derivatives of the pressure with respect to density and bulk modulus ($\partial p/\partial \rho$ and $\partial p/\partial \kappa$) (Tarantola, 1984). These derivatives, referred to as functional or Frechet derivatives in the literature, can be computed either by numerical differencing, a numerically intensive procedure, or by evaluation of very convenient analytical expressions (Pan, Phinney and Odom, 1986). These derivatives quantify the sensitivity of the model to perturbations in bulk modulus and density as a function of position. The two derivatives above are the most important. Other derivatives of interest can generally be constructed by application of the chain rule for differentiation. For example if we are interested in the sensitivity of the complex pressure field to perturbations in attenuation we can obtain it from $\partial p/\partial \kappa$ by making κ complex and then differentiating κ with respect to its imaginary part α .

In addition to quantifying the model and data variances and resolutions, it is also important to understand correlations between model parameters. For example, sound speed and layer thickness are correlated (Schmidt and Baggeroer, 1995). This directly affects how we should parameterize our model. Bube et al. (1995) have provided quantitative guidelines for how to discretize the model to compute inverse solutions which are as accurate as possible in the features of the model which are well determined (resolved) by travel time data. In particular the sound speed model should not be discretized much coarsely than the reflectors as a way of stabilizing the inverse problem, because that may force the computed layer depths to try to match aspects of the data which are caused by features in the sound speed field.

WORK COMPLETED

The first year task of coding the Frechet derivatives $\partial p / \partial \rho$ and $\partial p / \partial \kappa$ has been completed. The derivatives have been computed for a random bottom sediment model provided by John Goff of the Seabed Variability Group and by Jim Fulford of Bob Miyamoto's Capturing Uncertainty group. The relationships between the basic derivatives $\partial p / \partial \rho$ and $\partial p / \partial \kappa$ and sensitivities to other quantities of interest, e.g. sound speed and attenuation have been derived. The coding of the derivative for sensitivity to sediment shear is nearly complete.

If the source and receiver are assumed to be at the same level z in the medium, the Frechet derivatives are simple functions of the depth dependent Green's function. Consequently, it is possible to compute the Frechet derivatives from a single pass of any program, which computes the complete response of the medium. We have modified the wavenumber integration program OASES (Schmidt, 1988) to compute the derivatives. These are obtained as a direct byproduct of the acoustic field computation within the medium and have a negligible impact on the execution time of the program.

RESULTS

Results are illustrated in Figures 1-3. Figure 1 shows one realization of the sound speed and density for a sediment stack typical of the New Jersey shelf. The density has been scaled by a factor of 1000, so that density and sound velocity can be plotted on the same scale.

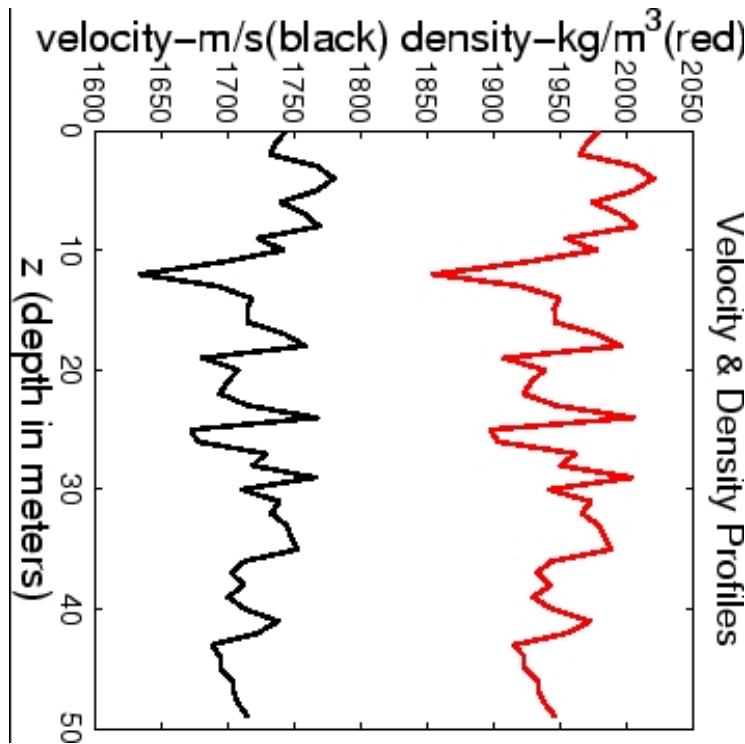


Figure 1. One realization of a sediment random velocity (black) and density (red) profile typical for the New Jersey shelf. Note that the density has been multiplied by 1000, so that density and velocity can be plotted on the same scale.

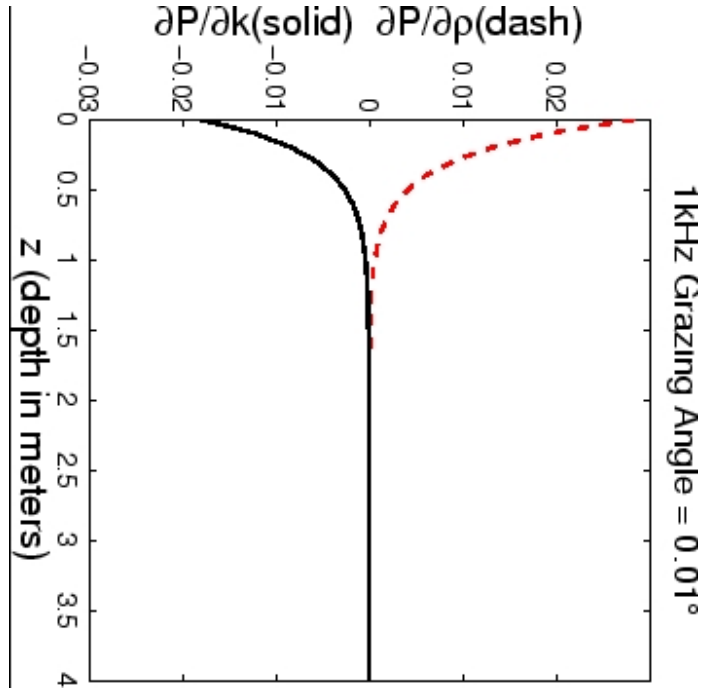


Figure 2. The Frechet derivatives $\partial p/\partial \rho$ (red dashed) and $\partial p/\partial \kappa$ (black solid) at a grazing angle of 0.01° . The frequency is 1kHz. Because the grazing angle is below critical, the field is evanescent in the bottom.

Figure 2 shows the Frechet derivatives at a grazing angle near 0° . Since this is below critical, the field in the bottom is purely evanescent, and essentially zero beyond a depth of about 1 meter. This means that a pressure measurement in the water is insensitive to any perturbations in density or bulk modulus for depths greater than one meter. Material properties of sediments at greater depths are completely unconstrained. However, because the derivatives for bulk modulus and density have opposite signs, perturbations in these two parameters will affect the measured pressure differently, and can be distinguished from one another.

Figure 3 shows the Frechet derivatives for normal incidence. The derivatives oscillate with depth, which reflects the oscillation of the Green's function itself. It is notable that at normal incidence, the derivatives for bulk modulus and density are the same. This is as it should be. The reflection coefficient and hence the pressure in the water is influenced by the acoustic impedance of the bottom. Perturbations in bulk modulus or density are indistinguishable from one another, and cannot be independently resolved with only normal incidence data. The derivatives at grazing angles intermediate between 0° and normal incidence exhibit properties intermediate between the two extremes illustrated by Figures 2 and 3. Of course the penetration depth, and therefore the sensitivity to perturbations at greater depths increases once the critical grazing angle is exceeded. This is obvious up comparing the penetration depth in Figure 1 with that in Figure 2.

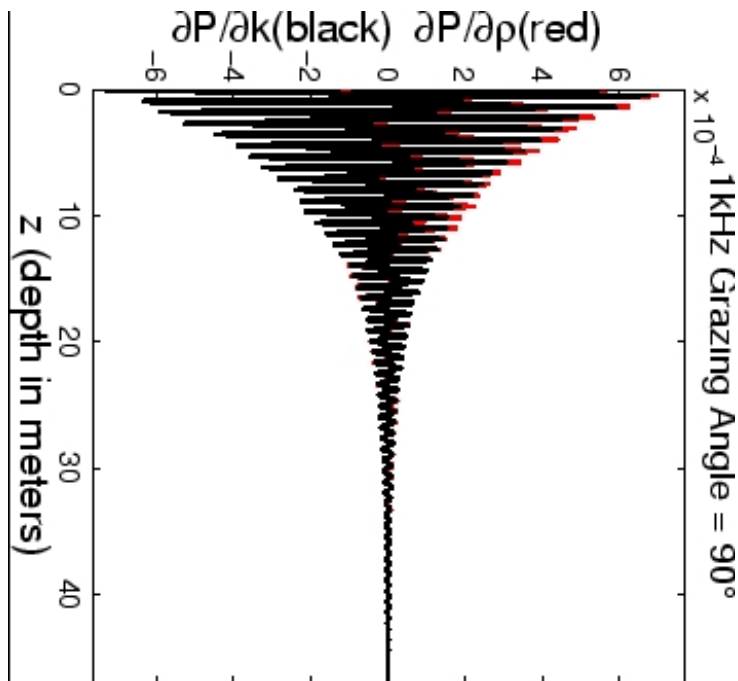


Figure 3. Frechet derivatives for density (red dashed) and bulk modulus (black solid) at normal incidence. The penetration depth is in excess of 20 meters, and since the derivatives are identical for the two parameters, perturbations of the two parameters cannot be distinguished from one another.

IMPACT/APPLICATIONS

Answering the questions posed in the Objectives section of this report will provide quantitative bounds on what can be expected from an optimal experiment designed for environmental characterization, how much we are giving up for a non-optimal experiment, and which environmental parameters are best and least determined and determinable.

TRANSITIONS

In the short term, the results of this research will be utilized by the other members of the Seabed Variability Team and the 6.2 Capturing Uncertainty team. In the longer term, the results of this research will permit the quantification of the effects of sampling density, scale variability, and parameter sensitivity for inclusion in seabed environmental databases.

RELATED PROJECTS

This research is directly related to the other sub-projects in the “Seabed Variability and its Influence on Acoustic Prediction Uncertainty” group.

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